

This following is intended to clarify some of the tasks—or “moves”—involved in the more problem-solving part of “doing mathematics.”

*Step 1:* Find an existing **problem**. This may be a problem you thought up last week, a problem created by a classmate, a problem given to you by your teacher, a problem you found in a book...

*Step 2:* Find an **answer** to your problem. (Easier said than done!) If this proves more difficult than expected, try solving a simpler case (or variation), getting help from a friend, doing some research, or just waiting for the muse to strike...

*Step 3:* Find a **solution** to your problem. Sound fishy? How is this different from Step 2? A solution *explains* and *justifies* your answer. My informing you that the volume of a unit tetrahedron is approximately 0.11785113 doesn't give you too much insight into *how* I went about figuring that out, nor does it say *why* you should believe such a statement. A solution is different from an answer in that someone unfamiliar with the problem at hand could read your solution and then solve a variation of the problem using the same solution strategy.

*Step 4:* Create a **variation** of your problem. This involves a tweak, a changing of a small detail, a varying of a case. A simple way of thinking about variations is that they usually involve changing one or more of the numbers in the problem. Another name for a variation is an *exercise*; it works the same muscles as your original problem and solidifies your understanding of and comfort with your own solution.

*Step 5:* Create a **generalization** of your problem. This involves setting up the problem in such a way that you can solve a number of cases at the same time. A common way of generalizing is replacing some or all of the numbers in a problem by variables. Often solutions to generalizations have much the same structure as the solution to the original problem.

*Step 6:* Create an **extension** of your problem. This involves changing the problem in some fundamental way while ideally still being able to use information obtained while solving the original problem. While the solution of your extension may not be obvious from your previous work, the experience and intuition you've gained by thinking through other similar problems should be helpful in guiding your thinking.

*Step 7:* Articulate **connections** between these problems that you've been working on and others you've worked on in the past. Notice similarities in their feel and their structure, as well as on your approaches to them and your solution methods. Try to figure out where your current problems lie in the greater mathematical landscape. No problem is an island.

***One more thing to keep in mind: these steps do not have to be done in any particular order (although sometimes it helps).***